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**ASSESSING HETEROGENEITY IN META-ANALYSIS:
Q STATISTIC OR I^2 INDEX?**

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ASSESSING HETEROGENEITY IN META-ANALYSIS: *Q* STATISTIC OR I^2 INDEX?

Abstract

In meta-analysis, the usual way of assessing whether a set of single studies are homogeneous is by means of the Q test. However, the Q test only informs us about the presence *versus* the absence of heterogeneity, but it does not report on the extent of such heterogeneity. Recently, the I^2 index has been proposed to quantify the degree of heterogeneity in a meta-analysis. In this paper, the performances of the Q test and the confidence interval around the I^2 index are compared by means of a Monte Carlo simulation. The results show the utility of the I^2 index as a complement to the Q test, although it has the same problems of power with a small number of studies.

KEY WORDS: Meta-analysis, effect size, heterogeneity, I^2 index, Monte Carlo method.

ASSESSING HETEROGENEITY IN META-ANALYSIS: *Q* STATISTIC OR I^2 INDEX?

In the last 25 years meta-analysis has been widely accepted in the social and health sciences as a very useful research methodology to quantitatively integrate the results of a collection of single studies on a given topic. In a meta-analysis the result of every study is quantified by means of an effect-size index (e.g., standardized mean difference, correlation coefficient, odds ratio, etc.) that can be applied to all studies, enabling us to give the study results in the same metric (Cooper, 1998; Cooper & Hedges, 1994; Egger, Smith, & Altman, 2001; Glass, McGaw, & Smith, 1981; Hedges & Olkin, 1985; Hunter & Schmidt, 2004; Rosenthal, 1991; Sutton, Abrams, Jones, Sheldon, & Song, 2000; Whitehead, 2002).

Typically, meta-analysis has three main goals: (a) to test whether the studies results are homogeneous, (b) to obtain a global index about the effect magnitude of the studied relation, joined to a confidence interval and its statistical significance, and (c) if there is heterogeneity among studies, to identify possible variables or characteristics moderating the results obtained. Here, we focus on how to assess the heterogeneity among the results from a collection of studies. Basically, there can be two sources of variability that explain the heterogeneity in a set of studies in a meta-analysis. One of them is the variability due to sampling error, also named within-study variability. The sampling error variability is always present in a meta-analysis, because every single study uses different samples. The other source of heterogeneity is the between-studies variability, which can appear in a meta-analysis when there is true heterogeneity among the population effect sizes estimated by the individual studies. The between-studies variability is due to the influence of an indeterminate number of characteristics that vary among the studies, such as those related to the characteristics of the samples, variations in the treatment, in the design quality, and so on (Brockwell & Gordon, 2001; Erez, Bloom, & Wells, 1996; Field, 2003; Hunter & Schmidt, 2000; National Research Council, 1992).

To assess the heterogeneity in meta-analysis is a crucial issue because the presence *versus* the absence of true heterogeneity (between-studies variability) can affect the statistical model that the meta-analyst decides to apply to the meta-analytic

database. So, when the studies' results only differ by the sampling error (homogeneous case) a fixed-effects model can be applied to obtain an average effect size. By contrast, if the study results differ by more than the sampling error (heterogeneous case), then the meta-analyst can assume a random-effects model, in order to take into account both within- and between-studies variability, or can decide to search for moderator variables from a fixed-effects model (Field, 2001, 2003; Hedges, 1994; Hedges & Olkin, 1985; Hedges & Vevea, 1998; Overton, 1998; Raudenbush, 1994).

The usual way of assessing whether there is true heterogeneity in a meta-analysis has been to use the Q test, a statistical test defined by Cochran (1954). The Q test is computed by summing the squared deviations of each study's effect estimate from the overall effect estimate, weighting the contribution of each study by its inverse variance. Under the hypothesis of homogeneity among the effect sizes, the Q statistic follows a chi-square distribution with $k - 1$ degrees of freedom, k being the number of studies. Not rejecting the homogeneity hypothesis usually leads the meta-analyst to adopt a fixed-effects model because it is assumed that the estimated effect sizes only differ by sampling error. In contrast, rejecting the homogeneity assumption can lead to applying a random-effects model that includes both within- and between-studies variability. A shortcoming of the Q statistic is that it has poor power to detect true heterogeneity among studies when the meta-analysis includes a small number of studies and excessive power to detect negligible variability with a high number of studies (Alexander, Scozzaro, & Borodkin, 1989; Cornwell, 1993; Cornwell & Ladd, 1993; Hardy & Thompson, 1998; Harwell, 1997; Osburn, Callender, Greener, & Ashworth, 1983; Paul & Donner, 1992; Sackett, Harris, & Orr, 1986; Sagie & Koslowsky, 1993; Sánchez-Meca & Marín-Martínez, 1997; Spector & Levine, 1987). Thus, a non-significant result for the Q test with a small number of studies can lead a reviewer to erroneously assume a fixed-effects model when there is true heterogeneity among the studies; and vice versa. On the other hand, the Q statistic does not inform us of the extent of true heterogeneity, only of its statistical significance.¹

¹ It is important to note that the low statistical power of the Q test for small number of studies has promoted the undesirable practice among some meta-analysts of ignoring the results of Q when it is not statistically significant, and searching for moderator variables. On the other hand, the meta-analyst can a priori adopt a statistical model (fixed- or random-effects model) on conceptual grounds. For example, if the meta-analyst wishes to generalize the meta-analytic results to a population of studies with similar characteristics than those of represented in the meta-analysis, a fixed-effects model can be selected. If, on

Another strategy for quantifying the true heterogeneity in a meta-analysis consists of estimating the between-studies variance, τ^2 . Assuming a random-effects model, the between-studies variance reflects how much the true population effect sizes estimated in the single studies of a meta-analysis differ. As the τ^2 depends on the particular effect metric used in a meta-analysis, it is not possible to compare the τ^2 values estimated from meta-analyses that have used different effect-size indices (e.g., standardized mean differences, correlation coefficients, odds ratios, etc.).

In order to overcome the shortcomings of the Q test and the τ^2 , Higgins and Thompson (2002; see also Higgins, Thompson, Deeks, & Altman, 2003) have proposed three indices for assessing heterogeneity in a meta-analysis: the H^2 , R^2 , and I^2 indices. As they are inter-related, here we focus on the I^2 index, because of its easy interpretation. The I^2 index measures the extent of true heterogeneity dividing the difference between the result of the Q test and its degrees of freedom ($k - 1$) by the Q value itself, and multiplied by 100. So, the I^2 index is similar to an intraclass correlation in cluster sampling (Higgins & Thompson, 2002). The I^2 index can be interpreted as the percentage of the total variability in a set of effect sizes due to true heterogeneity, that is, to between-studies variability. For example, a meta-analysis with $I^2 = 0$ means that all variability in effect size estimates is due to sampling error within studies. On the other hand, a meta-analysis with $I^2 = 50$ means that half of the total variability among effect sizes is caused not by sampling error, but by true heterogeneity between studies. Higgins and Thompson (2002) proposed a tentative classification of I^2 values with the purpose of helping to interpret its magnitude. Thus, percentages of around 25% ($I^2 = 25$), 50% ($I^2 = 50$), and 75% ($I^2 = 75$) would mean low, medium, and high heterogeneity, respectively. The I^2 index and the between-studies variance, τ^2 , are directly related: the higher the τ^2 , the higher the I^2 index. However, following Higgins and Thompson (2002), an advantage of the I^2 index in respect to τ^2 is that I^2 indices obtained from meta-analyses with different numbers of studies and different effect metrics are directly comparable.

the contrary, the meta-analytic results have to be generalized to a wider population of studies, a random-effects model should be the best option (Field, 2001; Hedges & Vevea, 1998).

Together with this descriptive interpretation of the I^2 index, Higgins and Thompson (2002) have derived a confidence interval for it that might be used in the same way as the Q test is used to assess heterogeneity in meta-analysis. Thus, if the confidence interval around I^2 contains the 0% value, then the meta-analyst can hold the homogeneity hypothesis. If, on the contrary, the confidence interval does not include the 0% value, then there is evidence for the existence of true heterogeneity. Using the I^2 index and its confidence interval is similar to applying the Q test. Because the I^2 index assesses not only heterogeneity in meta-analysis, but also the extent of that heterogeneity, it should be a more advisable procedure than the Q test in assessing whether or not there is true heterogeneity among the studies in a meta-analysis. However, the performance of the confidence interval around I^2 has not yet been studied in terms of the control of Type I error rate and statistical power.

The purpose of this paper is to compare, by a Monte Carlo simulation, the performance of the Q test and the confidence interval around the I^2 index, in terms of their control of Type I error rate and statistical power. Different effect-size indices were used and both the extent of true heterogeneity and the number of studies were varied. Thus, it is possible to test whether the confidence interval for I^2 overcomes the shortcomings of the Q test.

Effect-size indices

For each individual study, we assume two underlying populations representing the experimental *versus* control groups on a continuous outcome. Let μ_E and μ_C be the experimental and control population means, and σ_E and σ_C the population standard deviations, respectively. By including a control condition in the typical design we restrict the applicability of our results to research fields in which such designs make sense (e.g., treatment outcome evaluation in behavioral sciences, education, medicine, etc.). Under the assumptions of normal distributions and homoscedasticity, the usual parametric effect-size index is the standardized mean difference, δ , defined as the difference between the experimental and control population means, μ_E and μ_C , divided by the pooled population standard deviation, σ (Hedges & Olkin, 1985, p. 76, eq. 2),

$$\delta = \frac{\mu_E - \mu_C}{\sigma}. \quad (1)$$

The best estimator of the parametric effect size, δ , is the sample standardized mean difference, d , proposed by Hedges and Olkin (1985, p. 81, eq. 10) and computed by

$$d = c(m) \frac{\bar{y}_E - \bar{y}_C}{S}, \quad (2)$$

with \bar{y}_E and \bar{y}_C being the sample means of the experimental and control groups, respectively, and S being a pooled estimate of the within-group standard deviation, given by (Hedges & Olkin, 1985, p. 79),

$$S = \sqrt{\frac{(n_E - 1)S_E^2 + (n_C - 1)S_C^2}{n_E + n_C - 2}}, \quad (3)$$

with S_E^2 , S_C^2 , n_E , and n_C being the sample variances and the sample sizes of the experimental and control groups, respectively. The term $c(m)$ is a correction factor for the positive bias suffered by the standardized mean difference with small sample sizes and estimated by (Hedges & Olkin, 1985, p. 81, eq. 7),

$$c(m) = 1 - \frac{3}{4m - 1}, \quad (4)$$

with $m = n_E + n_C - 2$. The sampling variance of the d index is estimated by Hedges and Olkin (1985, p. 86, eq. 15) as

$$S_d^2 = \frac{n_E + n_C}{n_E n_C} + \frac{d^2}{2(n_E + n_C)}. \quad (5)$$

Another effect-size index from the d family is that proposed by Glass et al. (1981; see also Glass, 1976), consisting of dividing the difference between the

experimental and control group means by the standard deviation of the control group. Here we will represent this index by g (Glass et al., 1981, p. 105):²

$$g = c(m) \frac{\bar{y}_E - \bar{y}_C}{S_C}, \quad (6)$$

where S_C is the estimated standard deviation of the control group and $c(m)$ is the correction factor for small sample sizes given by equation (4), but with $m = n_C - 1$ (Glass et al., 1981, p. 113). The g index is recommended when the homoscedasticity assumption is violated. Glass et al. (1981) proposed dividing the mean difference by the standard deviation of the control group because the experimental manipulation can change the variability in the group; thus, under this circumstance they argue that it is better to estimate the population standard deviation by the control group standard deviation. Therefore, in the strict sense, the g index is estimating a different population effect size from that defined in equation (1), δ , consisting in dividing the mean difference by the population standard deviation of the control group: $\delta_C = (\mu_E - \mu_C)/\sigma_C$ (Glass et al., 1981, p. 112). The sampling variance of the g index is given by Rosenthal (1994, p. 238) as

$$S_g^2 = \frac{n_E + n_C}{n_E n_C} + \frac{g^2}{2(n_C - 1)}. \quad (7)$$

The statistical model

Once an effect-size estimate is obtained from each individual study, meta-analysis integrates them by calculating an average effect size, assessing the statistical heterogeneity around the average estimate, and searching for moderator variables when there is more heterogeneity than can be explained by chance. In general, the most realistic statistical model to integrate the effect estimates in a meta-analysis is the random-effects model, because it incorporates the two possible sources of heterogeneity

² Although Glass et al. (1981) represented this effect-size index with the Greek symbol Δ , here we prefer to keep Greek symbols to represent parameters, not estimates. Thus, we have selected the Latin letter g to represent this effect-size index.

among the studies in a meta-analysis: first, statistical variability caused by sampling error and, second, substantive variability.

Let T_i be the i th effect estimate in a collection of k studies ($i = 1, 2, \dots, k$). Here T_i corresponds to the d and g effect indices defined in Section 2 by equations (2) and (6), respectively. In a random-effects model it is assumed that every T_i effect is estimating a parametric effect size, θ_i , with conditional variance σ_i^2 , estimated by $\hat{\sigma}_i^2$. The estimated conditional variances, $\hat{\sigma}_i^2$, for the d and g indices proposed in Section 2 are defined by equations (5) and (7), respectively. The model can be formulated as $T_i = \theta_i + e_i$, where the errors, e_i , are normally and independently distributed with mean zero and variance σ_i^2 [$e_i \sim N(0, \sigma_i^2)$]. The conditional variance represents the within-study variability, that is, the variability produced by random sampling.

In turn, the parametric effect sizes θ_i pertain to an effect-parameter distribution with mean μ_0 and unconditional variance τ^2 . So, every θ_i parameter can be defined as $\theta_i = \mu_0 + u_i$, where it is usually assumed that the errors u_i are normally and independently distributed with mean zero and variance τ^2 [$u_i \sim N(0, \tau^2)$]. The unconditional variance, τ^2 , represents the extent of true heterogeneity among the study effects produced by the influence of an innumerable number of substantive (e.g., type of treatment, characteristics of the subjects, setting, etc.) and methodological (e.g., type of design, attrition, sample size, random *versus* non-random assignment, etc.) characteristics of the studies (Lipsey, 1994). Therefore, the random-effects model can be formulated as (Hedges & Vevea, 1998; Overton, 1998; Raudenbush, 1994):

$$T_i = \mu_0 + u_i + e_i, \quad (8)$$

where the errors u_i and e_i represent the two variability sources affecting the effect estimates, T_i , and quantified by the between-studies, τ^2 , and within-study, σ_i^2 , variances. Therefore, the effect estimates T_i will be normally and independently distributed with mean μ_0 and variance $\tau^2 + \sigma_i^2$ [$T_i \sim N(\mu_0, \tau^2 + \sigma_i^2)$].

When there is no true heterogeneity among the effect estimates, then the between-studies variance is zero ($\tau^2 = 0$), and there only will be variability due to sampling error, which is represented in the model by the conditional within-study variance, σ_i^2 . In this case, all the studies estimate one parametric effect size, $\theta_i = \theta$, and the statistical model simplifies to $T_i = \theta + e_i$, thus becoming a fixed-effects model. So, the fixed-effects model can be considered as a particular case of the random-effects model when there is no between-studies variability and, as a consequence, the effect estimates, T_i , are only affected by sampling error, σ_i^2 , following a normal distribution with mean θ (being in this case $\theta = \mu_0$) and variance σ_i^2 [$T_i \sim N(\theta, \sigma_i^2)$] for large sample sizes.

Assessing the extent of heterogeneity in a meta-analysis helps to decide which of the two models is the most plausible and this decision affects, at least, the weighting factor used to obtain an average effect size. The usual estimate of a mean effect size consists of weighting every effect estimate, T_i , by its inverse variance, w_i :

$$\bar{T} = \frac{\sum_i w_i T_i}{\sum_i w_i}. \quad (9)$$

In a fixed-effects model, the weighting factor for the i th study is estimated by $w_i = 1/\hat{\sigma}_i^2$. In a random-effects model, the weights are estimated by $w_i = 1/(\hat{\tau}^2 + \hat{\sigma}_i^2)$. For the d and g indices the estimated within-study variances, $\hat{\sigma}_i^2$, are defined in equations (5) and (7), respectively. A commonly used estimator of the between-studies variance, τ^2 , is an estimator based on the method of moments proposed by DerSimonian and Laird (1986):

$$\hat{\tau}^2 = \begin{cases} \frac{Q - (k - 1)}{c} & \text{for } Q > (k - 1) \\ 0 & \text{for } Q \leq (k - 1) \end{cases} \quad (10)$$

being c

$$c = \sum w_i - \frac{\sum w_i^2}{\sum w_i} \quad (11)$$

where w_i is the weighting factor for the i th study assuming a fixed-effects model ($w_i = 1/\hat{\sigma}_i^2$), k is the number of studies, and Q is the statistical test for heterogeneity proposed by Cochran (1954) and defined in equation (12). To avoid negative values for $\hat{\tau}^2$ when $Q \leq (k - 1)$, $\hat{\tau}^2$ is equated to 0. Note that due to this truncation, $\hat{\tau}^2$ is a biased estimator for τ^2 .

Assessing heterogeneity in meta-analysis

Quantifying the extent of heterogeneity among a collection of studies is one of the most troublesome aspects of a meta-analysis. It is important because it can affect the decision about the statistical model to be selected, fixed- or random-effects. On the other hand, if significant variability is found, potential moderator variables can be sought to explain this variability.

The between-studies variance, τ^2 , is the parameter in the statistical model that mainly represents the true (substantive, clinical) heterogeneity among the true effects of the studies. Therefore, a good procedure for determining whether there is true heterogeneity among a collection of studies should be positively correlated with τ^2 . At the same time, it should not be affected by the number of studies, and should be scale-free in order to be comparable among meta-analyses that have applied different effect-size indices.

The statistical test usually applied in meta-analysis for determining whether there is true heterogeneity among the studies' effects is the Q test, proposed by Cochran (1954) and defined as (Hedges & Olkin, 1985, p. 123, eq. 25):

$$Q = \sum w_i (T_i - \bar{T})^2, \quad (12)$$

where w_i is the weighting factor for the i th study assuming a fixed-effects model, and \bar{T} is defined in equation (9). If we assume that the conditional within-study variances, σ_i^2 , are known³, then under the null hypothesis of homogeneity ($H_0: \delta_1 = \delta_2 = \dots = \delta_k$; or also $H_0: \tau^2 = 0$), the Q statistic has a chi-square distribution with $k - 1$ degrees of freedom. Thus, Q values higher than the critical point for a given significance level (α) enable us to reject the null hypothesis and conclude that there is statistically significant between-study variation.

One problem with the Q statistic is that its statistical power depends on the number of studies, with power being very low or very high for a small or a large number of studies, respectively. To solve the problems of the Q statistic and the non comparability of the between-studies variance, τ^2 , among meta-analyses with different effect-size metrics, Higgins and Thompson (2002) have recently proposed the I^2 index. The I^2 index quantifies the extent of heterogeneity from a collection of effect sizes by comparing the Q value to its expected value assuming homogeneity, that is, to its degrees of freedom ($df = k - 1$):

$$I^2 = \begin{cases} \frac{Q - (k - 1)}{Q} \times 100\% & \text{for } Q > (k - 1) \\ 0 & \text{for } Q \leq (k - 1) \end{cases} \quad (13)$$

When the Q statistic is smaller than its degrees of freedom, then I^2 is truncated to zero. The I^2 index can easily be interpreted as a percentage of heterogeneity, that is, the part of total variation that is due to between-studies variance, $\hat{\tau}^2$. Therefore, there is a direct relationship between $\hat{\tau}^2$ and I^2 that can be formalized from the equations (10) and (13) as,

$$I^2 = \frac{c \hat{\tau}^2}{Q} \quad (14)$$

³ In practice, the population within-study variances never will be known, so they will have to be estimated from the sample data. For example, equations (5) and (7) are used to estimate the within-study variances for d and g indices.

To show empirically this relation, Figure 1 presents the results of a simulation, assuming a random-effects model with $\delta = 0.5$, $k = 50$, an average sample size $\bar{N} = 50$ ($n_E = n_C$ for every study), and manipulating the parametric between-studies variance, τ^2 , with values from 0.0 to 0.45, and 5 replications per condition. Figure 1 represents the obtained values of $\hat{\tau}^2$ and I^2 for every replication. So, for the manipulated conditions $\hat{\tau}^2$ values around 0.025, 0.05, and 0.15 correspond to I^2 values of 25%, 50%, and 75%, respectively. Further, note that beyond a certain value of τ^2 there is relatively little increase in I^2 . In particular, I^2 values higher than 85% will subsequently increase only slightly even if the between-studies variance increases substantially. Therefore, the I^2 index seems particularly useful in describing heterogeneity in a meta-analysis with a medium-to-low between-studies variance, and not so useful for large τ^2 values.

Higgins and Thompson (2002) have also developed a confidence interval for I^2 . The interval is formulated by calculating another of their proposed measures of heterogeneity, the H^2 index obtained by (Higgins & Thompson, 2002, p. 1545, eq. 6),

$$H^2 = \frac{Q}{k-1}, \quad (15)$$

also known as Birge's ratio (Birge, 1932). Then they define I^2 in terms of H^2 by means of (Higgins & Thompson, 2002, p. 1546, eq. 10),

$$I^2 = \frac{H^2 - 1}{H^2} \times 100\%. \quad (16)$$

This allows us to express inferences of H^2 in terms of I^2 . For practical application, Higgins and Thompson (2002, p. 1549) recommend a confidence interval for the natural logarithm of H , $\ln(H)$, assuming a standard normal distribution, that implies the Q statistic and k , given by,

$$\exp\{\ln(H) \pm |z_{\alpha/2}| \text{SE}[\ln(H)]\}, \quad (17)$$

where $|z_{\alpha/2}|$ is the $(\alpha/2)$ quantile of the standard normal distribution, and $\text{SE}[\ln(H)]$ is the standard error of $\ln(H)$ and is estimated by

$$SE[\ln(H)] = \begin{cases} \frac{1}{2} \frac{\ln(Q) - \ln(k-1)}{\sqrt{(2Q)} - \sqrt{(2k-3)}} & \text{if } Q > k \\ \sqrt{\left\{ \frac{1}{2(k-2)} \left(1 - \frac{1}{3(k-2)^2} \right) \right\}} & \text{if } Q \leq k \end{cases} \quad (18)$$

The confidence limits obtained by equation (15) are in terms of the H index. Consequently, they can be easily translated into the I^2 metric by applying equation (16) to both confidence limits.

An example will help to illustrate the calculations for the Q statistic and the I^2 index. Figure 2 presents some of the results of a meta-analysis about the effectiveness of delinquent rehabilitation programs (Redondo, Sánchez-Meca, & Garrido, 1999). In particular, Figure 2 presents the results of eight studies that compared a control group with one of two different correctional programs: three studies that compared a control group with a cognitive-behavioral treatment (CBT) and five studies that compared a control group with a therapeutic community program (TC). The comparisons were measured by the d index such as it is defined by equation (2). The purpose of the example is to illustrate the problems of the Q statistic and how the I^2 index is able to solve them.

As Figure 2 shows, the forest plot for the two groups of studies (the three studies for CBT and those for TC) reflect high heterogeneity in both cases, but heterogeneity is more pronounced for CBT studies than for TC studies. In fact, the estimated between-studies variance, $\hat{\tau}^2$, for CBT is clearly higher than for TC (0.24 and 0.06, respectively). However, the Q statistic is very similar and statistically significant in both cases [CBT: $Q(2) = 11.647$, $p = .003$; TC: $Q(4) = 11.931$, $p = .018$]. Thus, a direct comparison of the two Q values is not justified because their degrees of freedom differ, and can erroneously lead to the conclusion that the two groups of studies are similarly heterogeneous. But if we calculate the I^2 index for both groups, then differences in the extent of heterogeneity are clearly apparent: whereas CBT studies present an I^2 value of 82.8%, implying high heterogeneity, the TC studies present an I^2 value of medium size (66.5%). Thus, the I^2 index has been able to reflect differences in the degree of

heterogeneity between two groups of studies when the Q statistic offers very similar results for them.

The Q statistic is only useful for testing the existence of heterogeneity, but not the extent of heterogeneity. The I^2 index quantifies the magnitude of such heterogeneity and, if a confidence interval is calculated for it, then it can also be used for testing the heterogeneity hypothesis. In the example, the confidence limits obtained for the I^2 index applying equation (15) were for CBT studies from 47.6% to 94.4%, and for TC studies from 12.7% to 87.1%. In both cases, the 0% value is not contained by the confidence interval, showing the existence of heterogeneity and coinciding with the results obtained with the Q statistic. On the other hand, the width of the I^2 CI informs about the accuracy of the true heterogeneity estimation. Thus, as the number of CBT studies is higher than that of TC studies its true heterogeneity estimation is more accurate (confidence width = 46.8% and 74.4%, respectively). Therefore, the I^2 index with its confidence interval can substitute for the Q statistic, because it offers more information.

To further show the usefulness of the I^2 index to compare the extent of heterogeneity among different meta-analyses, Table 1 presents the results of four meta-analyses about treatment outcome in the social and behavioral sciences, in terms of their Q tests and I^2 indices. As every meta-analysis has a different number of studies (k), the Q values are not comparable. However, the I^2 indices enable to assess the extent of true heterogeneity as a percentage of total variation. So, for the three first meta-analyses their respective Q values only inform about the existence of heterogeneity, while the I^2 values allow us to identify the Sánchez-Meca et al. (1999) meta-analysis as showing the largest heterogeneity ($I^2 = 90.8\%$; 95% CI: 88.6% and 92.9%), in comparison to the other two ($I^2 = 67.3\%$, 95% CI: 57% and 75.2%; and $I^2 = 74.2\%$; 95% CI: 64.8% and 82.3%). On the other hand, the only meta-analysis with a nonsignificant Q test coincides with a $I^2 = 0\%$.

Method

The simulation study was programmed in GAUSS (Aptech Systems, 1992). For simulating each individual study, we have assumed a two-groups design (experimental *versus* control) and a continuous outcome. Two different effect-size indices, both

pertaining to the d metric, were defined: the standardized mean difference d index defined by Hedges and Olkin (1985) and the g index proposed by Glass et al. (1981). The main difference between them is the standard deviation used, as noted above.

To simulate a collection of k single studies we assumed a random-effects model. Thus, from a normal distribution of parametric effect sizes, θ_i , with mean $\mu_0 = 0.5$ and between-studies variance τ^2 [$\theta_i \sim N(0.5, \tau^2)$], collections of k studies were randomly generated. The mean effect-size parameter was fixed at $\mu_0 = 0.5$, as it can be considered an effect of medium magnitude (Cohen, 1988).⁴ Once a θ_i value was randomly selected, two distributions (for the experimental and control groups) were generated, with means $\mu_E = \theta_i$ and $\mu_C = 0$, variance for the control group equal to 1 ($\sigma_C^2 = 1$) and variance for the experimental group equal 1, 2, or 4 ($\sigma_E^2 = 1, 2, \text{ or } 4$), depending on the ratio between σ_E^2 and σ_C^2 . The distributions for scores in experimental and control groups might be normal or non-normal, with different values of skewness and kurtosis in the non-normal cases. Then, two random samples (experimental and control) were selected from the two distributions with sizes $n_E = n_C$, and the means (\bar{y}_E and \bar{y}_C) and standard deviations (S_E and S_C) were obtained. Thus, the standardized mean differences, d (eq. 2) and g (eq. 6), and their sampling variances, S_d^2 (eq. 5) and S_g^2 (eq. 7), were calculated. The calculations for the d and g indices, and their sampling variances, were repeated for each one of the k studies of each simulated meta-analysis. Then, for every set of effect estimates (d and g indices), the calculations to obtain the Q statistic with its statistical significance and the I^2 index with its confidence interval were carried out, applying equations (11), (12), and (15), respectively. Thus, the following factors were manipulated in the simulations:

- (a) The between-studies variance, τ^2 , with values 0, 0.04, 0.08, and 0.16. When $\tau^2 = 0$, the statistical model becomes a fixed-effects model, because there is no between-studies variance. The selected values of τ^2 were similar to those used in other simulation studies (Biggerstaff & Tweedie, 1997; Brockwell & Gordon, 2001; Erez et al., 1996; Field, 2001; Hedges & Vevea, 1998; Overton, 1998).

⁴ Additional simulations varying the value of μ_0 showed similar results to that of $\mu_0 = 0.5$ for the Q statistic and the I^2 index. Thus, we maintained fixed μ_0 to simplify the simulation design.

- (b) The number of studies for each meta-analysis, k , with values 5, 10, and 20. These values for k are common in real meta-analyses and they were selected to study the performance of Q and I^2 when the number of studies is small, because the literature suggests poor performance under these conditions (Hardy & Thompson, 1998; Harwell, 1997; Sánchez-Meca & Marín-Martínez, 1997).
- (c) The within-study variances for experimental and control groups were varied using ratios for experimental and control groups, respectively, of 1:1, 2:1, and 4:1 as suggested in the literature (e.g., McWilliams, 1991; Wilcox, 1987). The variance of the experimental group was increased in comparison to that of the control group because increases in variability are more plausible when there is experimental manipulation (e.g., a psychological treatment) (Glass et al., 1981).
- (d) Usually, the studies integrated in a meta-analysis have different sample sizes. Thus, the mean sample size for each generated meta-analysis was varied with values $\bar{N} = 30, 50$, and 80. The sample-size distribution used in the simulations was obtained by a review of the meta-analyses published in 18 international psychological journals. This review enabled us to obtain a real sample-size distribution characterized by a Pearson skewness index of +1.464 (more detailed information is given in Sánchez-Meca & Marín-Martínez, 1998). In accord with this value, three vectors of five N s each were selected averaging 30, 50, or 80, with the skewness index given above to approximate real data: [12, 16, 18, 20, 84], [32, 36, 38, 40, 104], and [62, 66, 68, 70, 134]. Each vector of N s was then replicated either 2 or 4 times for meta-analyses of $k = 10$ and 20 studies, respectively. The within-study sample sizes for the experimental and control groups were equal ($n_E = n_C$, being $N = n_E + n_C$, for each single study). For example, the sample sizes vector [12, 16, 18, 20, 84] means that the experimental and control group sample sizes were, respectively, [$n_E = n_C = 6, 8, 9, 10, 42$].
- (e) Scores for the experimental and control participants in each pseudo-study were generated assuming a variety of different distributions: both normal distributions and non-normal distributions. To generate non-normal distributions, the normality pattern was manipulated to obtain skewed distributions by means of the Fleishman (1978) algorithm, with the following values of skewness/kurtosis: 0.5/0, 0.75/0, and 1.75/3.75. These values of

skewness and kurtosis can be considered of a moderate magnitude (DeCarlo, 1997; Hess, Olejnik, & Huberty, 2001).

To simplify the design of the simulation study we did not cross all of the manipulated factors. In the condition of normal distributions for the experimental and control groups in the single studies, we crossed all the factors mentioned above, obtaining a total of 4 (τ^2 values) \times 3 (k values) \times 3 (variance ratios) \times 3 (\bar{N} values) = 108 conditions. For the three conditions in which the score distributions of the single studies were non-normal, the design of the simulation was simplified by reducing the number of studies in each meta-analysis to only two conditions: $k = 5$ and 20. Thus, the number of conditions was 3 (τ^2 values) \times 2 (k values) \times 3 (variance ratios) \times 3 (\bar{N} values) \times 3 (non-normal distributions) = 162. Therefore, the total number of manipulated conditions was 108 (normal distributions) + 162 (non-normal distributions) = 270 conditions. For each of the 270 conditions, 10,000 replications were generated. To obtain estimates of the Type I error rate and statistical power for the Q statistic and the confidence interval for the I^2 index, assuming a significance level of $\alpha = .05$, the following computations over the 10,000 replications in each condition were carried out,

- (a) In conditions where the between-studies variance was zero ($\tau^2 = 0$), the proportion of false rejections of the null hypothesis of homogeneity in the 10,000 replications was the empirical Type I error rate for the Q statistic. Similarly, the proportion of replications in which the confidence interval for I^2 did not contain the value $\tau^2 = 0$ represented its empirical Type I error rate. Following Cochran (1952) we assumed that good control of the Type I error rate for $\alpha = 0.05$ implies empirical rates in the range 0.04-0.06.
- (b) In conditions with non zero between-studies variance ($\tau^2 > 0$), the proportion of rejections of the homogeneity hypothesis was the empirical power for the Q statistic, and the proportion of replications in which the confidence interval for I^2 did not contain the value $\tau^2 = 0$, was an estimate of the power of this procedure. Following Cohen (1988), we adopted 0.80 as the minimum advisable power.

Results

First, we will present the results obtained through the manipulated conditions in respect to the control of Type I error rates achieved by the Q test and the confidence interval of I^2 (I^2 CI) both for the d and g indices. Then, the results in terms of statistical power will be shown.⁵

Type I error rate.

Estimated Type I error rates were obtained when the between-studies variance was zero ($\tau^2 = 0$). For each condition, the Type I error rate was calculated dividing by 10,000 the number of replications in which the null hypothesis was incorrectly rejected using the Q test, or the number of replications in which the value zero was not in the I^2 CI. Figure 3 presents results for Type I error rates as a function of the number of studies and the average sample size under the conditions assuming normality and homoscedasticity in the experimental and control groups' distributions. As Figure 3 shows, good control of the Type I error rate is achieved with both the Q test and the I^2 CI when the d index is used, but not with the g index. The good control of the Type I error for Q and I^2 CI with the d index is neither affected by the number of studies nor by the average sample size in the meta-analysis. However, note that the Type I error rate for I^2 CI with the d index is slightly lower than the .04 limit that we have assumed as representing a good adjustment to the .05 nominal significance level. On the other hand, with the g index, Q and I^2 CI present Type I error rates clearly higher than the nominal $\alpha = .05$, and importantly above the .06 limit. This poor performance slightly increases with the number of studies, but diminishes with the average sample size.

When the experimental- and control-group distributions were normal but the homoscedasticity assumption was not met, both Q and I^2 CI maintained good control of the Type I error rate with the d index (although the Type I error rate for I^2 CI being slightly under the .04 limit). This result was not affected by the number of studies and the average sample size, as Figure 4 shows. However, with the g index, a dramatic increase of the Type I error rate for Q and I^2 CI was found as the ratio between

experimental and control groups' variances was increased. As Figure 4 shows, the poor performance of Q and I^2 CI for the g index is affected by the number of studies and the average sample size, with trends similar to those obtained assuming normality and homoscedasticity.

When the experimental- and control-group distributions were non-normal and the homoscedasticity assumption was met, the control of the Type I error rate was good for both the Q test and I^2 CI computed for the d index. However, as the distributions deviated from normality, the Type I error rates of Q and the I^2 CI for the g index suffered a drastic increase. Finally, when the normality and homoscedasticity assumptions were not met, the Type I error rates of Q and I^2 CI for the d index maintained their proximity to the nominal $\alpha = .05$, whereas the performance of Q and I^2 CI for the g index remained very poor (see Figure 5).

Statistical power.

The estimated power values were obtained when between-studies variance was higher than zero ($\tau^2 > 0$). For each condition, the power value was calculated by dividing by 10,000 the number of replications in which the null hypothesis is correctly rejected using the Q test, or the number of replications in which the zero value was not in the confidence interval of I^2 .

Figure 6 shows the estimated power values when the normality and homoscedasticity assumptions were met, as a function of the number of studies and the between-studies variance. As expected, the estimated power for all of the procedures increased as the number of studies and the between-studies variance increased. The results also showed that the recommended 0.8 power value (Cohen, 1988) was reached only when there were 20 or more studies and a large between-studies variance ($\tau^2 \geq 0.16$). Similar power results were obtained as a function of the average sample size.

With normal distributions and heteroscedastic variances the power values for Q and I^2 CI showed similar trends as a function of the number of studies: the higher the number of studies the higher the power (see Figure 7). Although the trend was similar

⁵ Because of space limitations, not all of the tables and figures for all of the manipulated conditions are

for all of the procedures, Q and I^2 CI achieved a higher power when the g index was used in comparison with the d index. The better power obtained with the g index under heterogeneous variances occurred because g uses the control group standard deviation, whereas the d index uses a pooled standard deviation obtained from the experimental and control groups. In our simulations we assumed, as Glass suggested (Glass et al., 1981), control-group standard deviations smaller than those of the experimental groups. This circumstance leads to higher heterogeneity among g indices than among d indices. As a consequence, it is easier for Q and I^2 CI to detect heterogeneity among g indices. Finally, similar power results were obtained when the normality and homoscedasticity assumptions were not met. As Figure 8 shows, Q and I^2 CI achieved higher power values with the g index than with the d index. However, the inflated Type I error rates obtained with the g index implies an inappropriate performance of Q and I^2 CI with this index.

Discussion

Traditionally, the Q test has been the normal procedure in assessing the heterogeneity hypothesis in meta-analysis (Cooper & Hedges, 1994). Recently, a new statistic named I^2 , and a confidence interval around it, has been proposed to estimate the extent of heterogeneity, as well as its statistical significance (Higgins & Thompson, 2002; Higgins et al., 2003). Assessing heterogeneity in meta-analysis is a crucial issue because the meta-analyst's decision to select the statistical model to be applied in a meta-analysis (fixed- versus random-effects model) can be affected by the result of a homogeneity test. Due to the importance of this issue, the purpose of this paper was to compare the performance of two procedures, the Q test and I^2 CI, to assess the heterogeneity among a set of single studies in a meta-analysis. In particular, Type I error rates and statistical power of the two procedures were examined by means of Monte Carlo simulation as a function of the number of studies, the average sample size, the between-studies variance, and the normality and homoscedasticity of the experimental- and control-group distributions. On the other hand, two different effect-size indices pertaining to the d family were used to calculate the Q test and the I^2 CI: d and g indices. A comparison between the Q test and the I^2 CI has not yet been carried out.

Therefore, the results of our study cast some light on the performance of both procedures in assessing heterogeneity in a meta-analysis.

The results of the simulation study helped us reach several conclusions related to our goals. In respect to the control of Type I error rate, the performance of the Q test and the I^2 CI was very similar. In fact, there were more differences between the procedures based on d and g indices than between the Q test and the I^2 CI. In particular, with the d index both procedures achieved good control of the Type I error rate, whereas the performance of the Q test and the I^2 CI calculated with the g index was very poor. On the other hand, Type I error rates for both procedures with the d index were not affected by the number of studies and the average sample size. However, the performance of the Q test and the I^2 CI depend on the effect-size metric. Therefore, confidence intervals around I^2 obtained from meta-analyses with different effect-size metrics should be interpreted cautiously, because they may not be comparable.

In respect to statistical power, there were no notable differences between the Q test and the I^2 CI. As expected, both procedures exhibited higher power as the number of studies, the average sample size, and the between-studies variance increased. However, with a small number of studies ($k < 20$) and/or average of sample size ($\bar{N} < 80$), the power is under the minimum advisable value 0.8. In fact, both procedures calculated with the d index reached power values as small as 0.3 in some conditions. Therefore, the I^2 CI suffers the same problem as the Q test in terms of statistical power.

On the other hand, the power of these procedures calculated with the g index was higher than that obtained with the d index. However, the highest power for summaries of the g index was achieved at the expense of an inadmissibly large Type I error rate. Therefore, the performance of the Q test and I^2 CI with the g index is poor. In any case, the usefulness of our results for the g index should be limited to real meta-analyses where systematically the variability in the experimental groups is higher than that of the control groups; this only will happen when the implementation of a treatment produces an overdispersion of the subject scores in comparison to the control group scores. The poor Type I error performance of the Q test and the I^2 index with g index under normality and homoscedasticity raises various concerns, including the accuracy of the sampling variance of this index. Our results also show a negligible effect on the

Type I error rates and statistical power of the Q test and the I^2 CI with the d index when the usual assumptions about the experimental- and control-group distributions (normality and homoscedasticity) are not met.

In summary, our findings show that the I^2 CI performs in a similar way to the Q test from an inferential point of view. But the I^2 index has important advantages in respect to the classical Q test. First, it is easily interpretable because it is a percentage and does not depend on the degrees of freedom. Another advantage is that it provides a way of assessing the magnitude of the heterogeneity in a meta-analysis, whereas the Q test reports about the statistical significance of the homogeneity hypothesis. On the other hand, the I^2 CI informs about the accuracy of the true heterogeneity estimation.

In addition, the I^2 index can be used to assess the degree of misspecification error when a qualitative moderator variable is tested. In particular, for every category of the moderator variable, an I^2 index can be calculated and their values are directly compared in order to determine which categories show a good fit to the statistical model and which ones do not. On the other hand, the I^2 index can be useful to compare the fitting of alternative models with different moderator variables regardless of their degrees of freedom. Future research in this area can help to ascertain the usefulness of the I^2 index when the statistical model in a meta-analysis includes moderator variables.

Some warnings for the use of the I^2 index have to be taken into account. The confidence interval around I^2 used to assess the homogeneity hypothesis in meta-analysis suffers the same problems of low power that the Q test does when the number of studies is small. The I^2 CI does not solve the shortcomings of the Q test. Therefore, using either the I^2 CI or the Q test to decide upon the statistical model (fixed- *versus* random-effects model) in a meta-analysis can be misleading. With a small number of studies ($k < 20$) both the I^2 CI and the Q test should be interpreted very cautiously.

As the I^2 index and its confidence interval allow us to assess simultaneously both the statistical significance and the extent of heterogeneity, the meta-analyst can obtain a more complete picture of heterogeneity than that offered by the Q test. Therefore, we propose using I^2 and its confidence interval to assess heterogeneity in meta-analysis,

although taking into account its low statistical power when the number of studies is small.

On the other hand, our results comparing the d and g indices have shown very different performances for the I^2 CI depending on the effect-size metric. Under our manipulated conditions, the g index systematically showed an inappropriate control of the Type I error rate and, therefore, using the Q test or the I^2 CI with this index is unadvisable. However, the poor performance that we have found for the Q test and the I^2 CI with the g index is only applicable when the studies systematically present a higher variability in the experimental group than in the control group. More research should be carried out to study the comparability of the I^2 index with other effect-size metrics, such as correlation coefficients, odds ratios, and so on. Finally, it should be noted that the results of our study are limited to the simulated conditions. Consequently, additional research efforts manipulating other factors, or examining different levels of these factors, can help to assess the generalizability of our findings.

References

- Alexander, R. A., Scozzaro, M. J., & Borodkin, L. J. (1989). Statistical and empirical examination of the chi-square test for homogeneity of correlations in meta-analysis. *Psychological Bulletin*, 106, 329-331.
- Aptech Systems, Inc. (1992). *The GAUSS system* (Vers. 3.0). Kent, WA: Author.
- Biggerstaff, B. J., & Tweedie, R. L. (1997). Incorporating variability estimates of heterogeneity in the random effects model in meta-analysis. *Statistics in Medicine*, 16, 753- 768.
- Birge, R. T. (1932). The calculation of errors by the method of least squares. *Physical Review*, 40, 207-227.
- Brockwell, S. E., & Gordon, R. I.(2001) A comparison of statistical methods for meta-analysis. *Statistics in Medicine*, 20, 825- 840.
- Cochran, W. G. (1952). The χ^2 test of goodness of fit. *Annals of Mathematical Statistics*, 23, 315-345.
- Cochran, W. G. (1954). The combination of estimates from different experiments. *Biometrics*, 10, 101-129.

- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). New York: Academic Press.
- Cooper, H. (1998). *Integrating research: A guide for literature reviews* (3rd ed.). Newbury Park, CA: Sage.
- Cooper, H., & Hedges, L. V. (Eds.). (1994). *The handbook of research synthesis*. New York: Russell Sage Foundation.
- Cornwell, J. M. (1993). Monte Carlo comparison of three tests for homogeneity of independent correlations. *Educational & Psychological Measurement*, 53, 605-618.
- Cornwell, J. M., & Ladd, R. T. (1993). Power and accuracy of the Schmidt and Hunter meta-analytic procedures. *Educational & Psychological Measurement*, 53, 877-895.
- DeCarlo, L. T. (1997). On the meaning and use of kurtosis. *Psychological Methods*, 2, 292-306.
- DerSimonian, R., & Laird, N. (1986) Meta-analysis in clinical trials. *Controlled Clinical Trials*, 7, 177-188.
- Egger, M., Smith, G. D., & Altman, D. G. (Eds.). (2001). *Systematic reviews in health care: Meta-analysis in context* (2nd ed.). London: BMJ Publishing Group.
- Erez, A., Bloom, M. C., & Wells, M. T. (1996). Using random rather than fixed effects models in meta-analysis: Implications for situational specificity and validity generalization. *Personnel Psychology*, 49, 275-306.
- Field, A. P. (2001). Meta-analysis of correlation coefficients: A Monte Carlo comparison of fixed- and random-effects methods. *Psychological Methods*, 6, 161-180.
- Field, A. P. (2003). The problems in using fixed-effects models of meta-analysis on real-world data. *Understanding Statistics*, 2, 77-96.
- Fleishman, A. I. (1978). A method for simulating nonnormal distributions. *Psychometrika*, 43, 521-531.
- Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational Research*, 5, 3-8.
- Glass, G. V., McGaw, B., & Smith, M. L. (1981). *Meta-analysis in social research*. Newbury Park, CA: Sage.
- Hardy, R. J., & Thompson, S. G. (1996). A likelihood approach to meta-analysis with random effects. *Statistics in Medicine*, 15, 619-629.

- Hardy, R. J., & Thompson, S. G. (1998). Detecting and describing heterogeneity in meta-analysis. *Statistics in Medicine*, 17, 841-856.
- Harwell, M. (1997). An empirical Study of Hedge's homogeneity test. *Psychological Methods*, 2, 219-231.
- Hedges, L. V. (1994). Fixed effects models. In H. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 285-299). New York: Russell Sage Foundation.
- Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis*. Orlando, FL: Academic Press.
- Hedges, L. V., & Vevea, J. L. (1998). Fixed- and random-effects models in meta-analysis. *Psychological Methods*, 3, 486-504.
- Hess, B., Olejnik, S., & Huberty, C. J. (2001). The efficacy of two improvement-over-chance effect sizes for two-group univariate comparisons under variance heterogeneity and nonnormality. *Educational & Psychological Measurement*, 61, 909-936.
- Higgins, J. P. T., & Thompson, S. G. (2002). Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine*, 21, 1539-1558.
- Higgins, J. P. T., Thompson, S. G., Deeks, J. J., & Altman, D. G. (2003). Measuring inconsistency in meta-analyses. *British Medical Journal*, 327, 557-560.
- Hunter, J. E., & Schmidt, F. L. (2000). Fixed effects vs random effects meta-analysis models: Implications for cumulative research knowledge. *International Journal of Selection & Assessment*, 8, 275-292.
- Hunter, J. E., & Schmidt, F. L. (2004). *Methods of meta-analysis: Correcting error and bias in research findings* (2nd ed.). Newbury Park, CA: Sage.
- Lipsey, M. W. (1994). Identifying potentially interesting variables and analysis opportunities. In H. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 111-123). New York: Russell Sage Foundation.
- McWilliams, L. (1991, April). *Variance heterogeneity in empirical studies in education and psychology*. Paper presented at the annual colloquium of the American Educational Research Association, San Francisco.
- Moreno, P. J., Méndez, F. X., & Sánchez-Meca, J. (2001). Effectiveness of cognitive-behavioural treatment in social phobia: A meta-analytic review. *Psychology in Spain*, 5, 17-25.

- National Research Council. (1992). *Combining information: Statistical issues and opportunities for research*. Washington, DC: National Academy Press.
- Osburn, H. G., Callender, J. C., Greener, J. M., & Ashworth, S. (1983). Statistical power of tests of the situational specificity hypothesis in validity generalization studies: A cautionary note. *Journal of Applied Psychology*, 68, 115-122.
- Overton, R. C. (1998). A comparison of fixed-effects and mixed (random-effects) models for meta-analysis tests of moderator variable effects. *Psychological Methods*, 3, 354-379.
- Paul, S. R., & Donner, A. (1992). Small sample performance of tests of homogeneity of odds ratios in K 2x2 tables. *Statistics in Medicine*, 11, 159-165.
- Raudenbush, S. W. (1994). Random effects models. In H. Cooper and L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 301-321). New York: Russell Sage Foundation.
- Redondo, S., Garrido, V., & Sánchez-Meca, J. (1997). What works in correctional rehabilitation in Europe: A meta-analytic review. In S. Redondo, V. Garrido, J. Pérez, & R. Barberet (Eds.), *Advances in psychology and law: International contributions* (pp. 499-523). Berlin: De Gruyter.
- Redondo, S., Sánchez-Meca, J., & Garrido, V. (1999). The influence of treatment programs on the recidivism of juvenile and adult offenders: An European meta-analytic review. *Psychology, Crime & Law*, 5, 251-278.
- Rosenthal, R. (1991). *Meta-analytic procedures for social research* (rev. ed.). Newbury Park, CA: Sage.
- Rosenthal, R. (1994). Parametric measures of effect size. In H. Cooper and L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 231-244). New York: Russell Sage Foundation.
- Sackett, P. R., Harris, M. M., & Orr, J. M. (1986). On seeking moderator variables in the meta-analysis of correlational data: A Monte Carlo investigation of statistical power and resistance to Type I error. *Journal of Applied Psychology*, 71, 302-310.
- Sagie, A., & Koslowsky, M. (1993). Detecting moderators with meta-analysis: An evaluation and comparison of techniques. *Personnel Psychology*, 46, 629-640.
- Sánchez-Meca, J., & Marín-Martínez, F. (1997). Homogeneity tests in meta-analysis: A Monte Carlo comparison of statistical power and Type I error. *Quality & Quantity*, 31, 385-399.

- Sánchez-Meca, J., & Marín-Martínez, F. (1998). Weighting by inverse variance or by sample size in meta-analysis: A simulation study. *Educational & Psychological Measurement*, 58, 211-220.
- Sánchez-Meca, J., Olivares, J., & Rosa, A.I. (1999). The problem of tobacco addiction: Meta-analysis of behavioural treatments in Spain. *Psychology in Spain*, 3, 36-45.
- Shadish, W. R., & Haddock, C. K. (1994). Combining estimates of effect size. In H. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 261-281). New York: Russell Sage Foundation.
- Spector, P. E., & Levine, E. L. (1987). Meta-analysis for integrating study outcomes: A Monte Carlo study of its susceptibility to Type I and Type II errors. *Journal of Applied Psychology*, 72, 3-9.
- Sutton, A. J., Abrams, K. R., Jones, D. R., Sheldon, T. A., & Song, F. (2000). *Methods for meta-analysis in medical research*. Chichester, UK: Wiley.
- Whitehead, A. (2002). *Meta-analysis of controlled clinical trials*. Chichester, UK: Wiley.
- Wilcox, R. R. (1987). New designs in analysis of variance. *Annual Review of Psychology*, 38, 29-60.

Table 1. Q tests and I^2 indices for several meta-analyses.

Source	Issue	k	Q	p	I^2	$\frac{95\% \text{ CI}}{L_l - L_u}$
Redondo, Garrido, & Sánchez-Meca (1997)	Correctional treatment outcome	57	171.27	<.0001	67.3%	56.96% - 75.16%
Redondo, Sánchez-Meca, & Garrido (1999)	Correctional treatment outcome	32	124.07	<.0001	74.2%	64.82% - 82.26%
Sánchez-Meca, Olivares, & Rosa (1999)	Tobacco addiction treatment outcome	36	389.07	<.0001	90.8%	88.55% - 92.93%
Moreno, Méndez, & Sánchez-Meca (2000)	Social phobia treatment outcome	39	19.163	>.05	0%	-

k : number of studies. Q : homogeneity test. p : probability level associated to the Q test. I^2 : I^2 index. 95% CI: 95% confidence interval around I^2 . L_l and L_u : lower and upper confidence limits for I^2 .

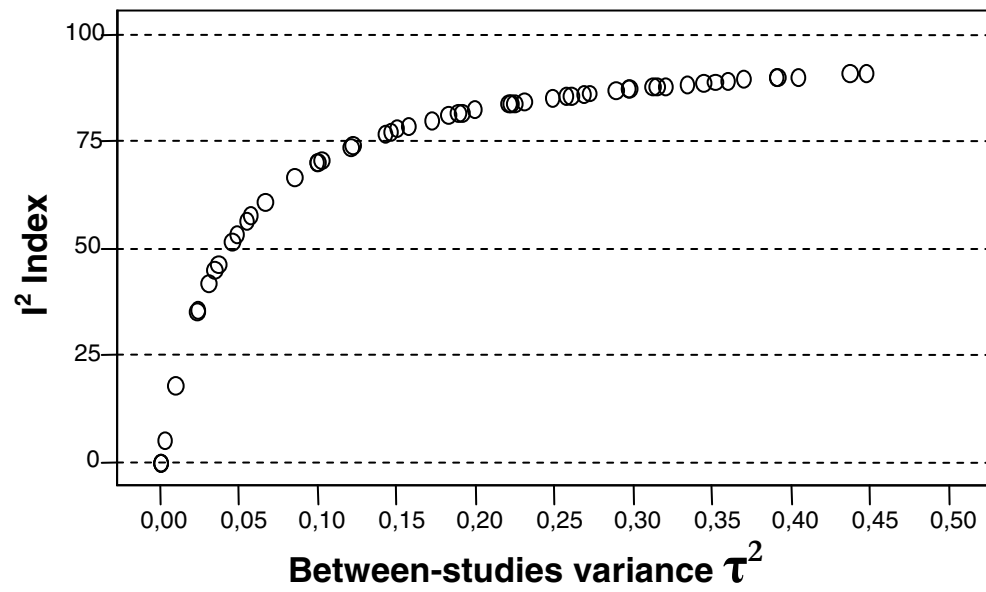


Figure 1. Results of the simulation relating I^2 values to estimated between-studies variance.

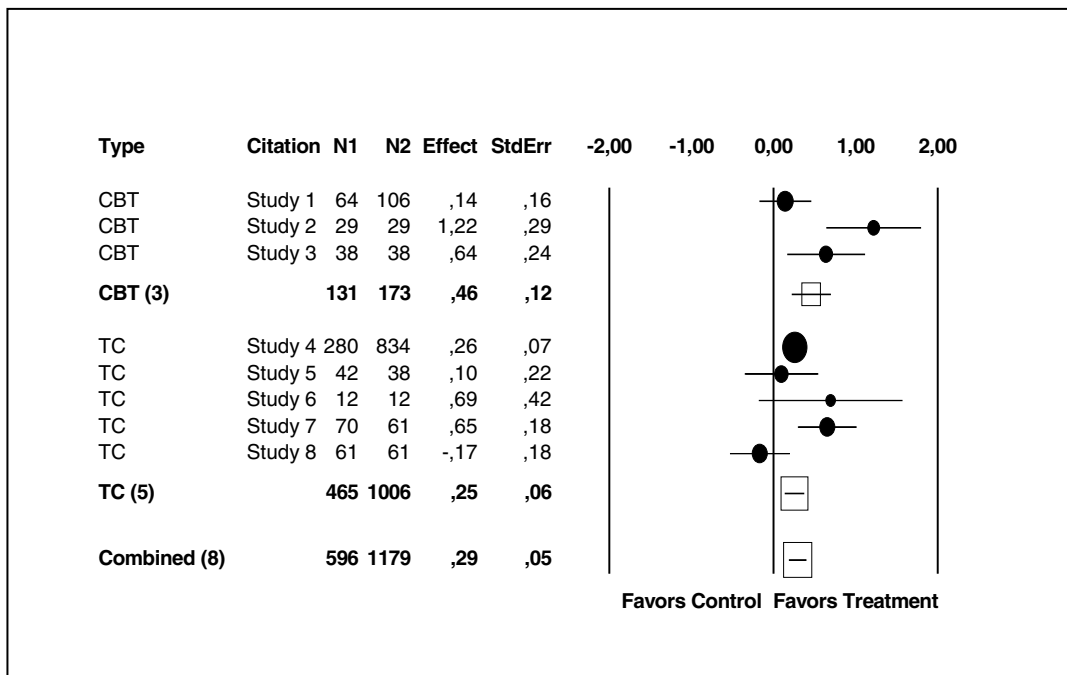


Figure 2. Forest plot of three studies for cognitive-behavioral treatment (CBT) and five studies for therapeutic community (TC). N1 and N2: sample sizes for treatment and control groups, respectively. Effect: standardized mean difference following equation (2) (d index). StdErr: standard error of the d index, obtained by calculating the square root of the equation (5). Filled circles represent the individual effect size for every study; the boxes refer to the average effect sizes for CBT studies, TC studies, and all of the studies; the horizontal lines around the circles and boxes indicate the width of confidence intervals; and the central vertical line represents the null effect size.

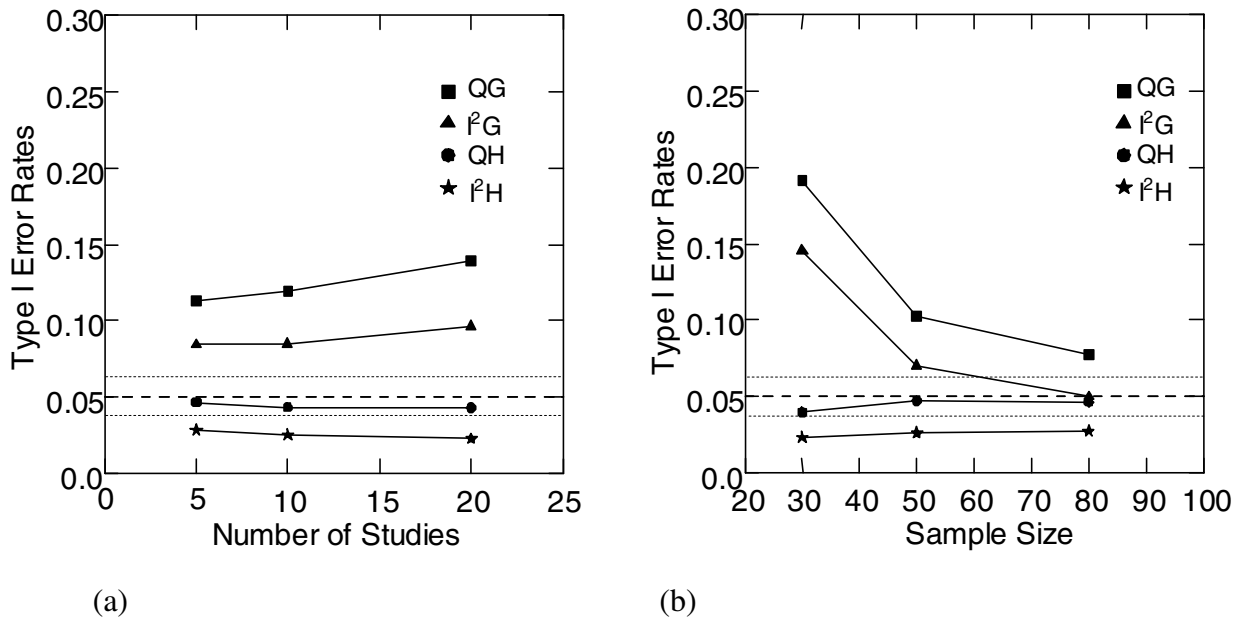


Figure 3. Type I error rates when normality and homogenous variances for experimental and control groups are assumed as a function of: (a) the number of studies, and (b) the sample size, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 with d by Hedges (I^2H), and g by Glass (I^2G) indexes.

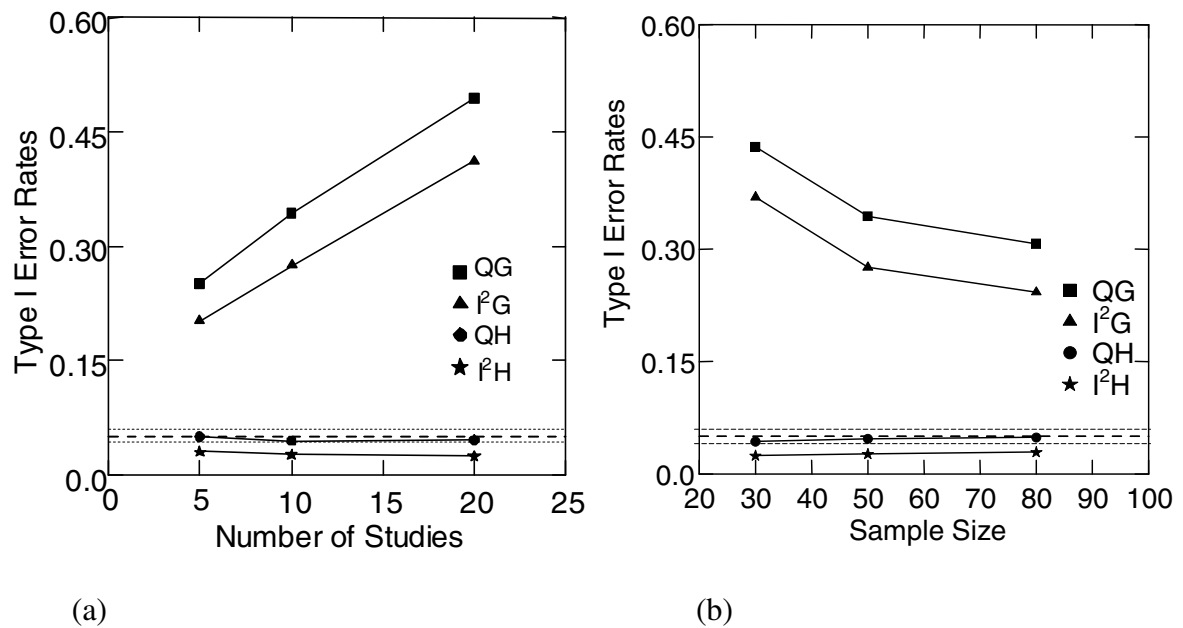


Figure 4. Type I error rates when normality and heterogeneous variances for experimental and control groups (ratio EG:CG = 2:1) are assumed as a function of: (a) the number of studies and (b) the sample size, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 using d index by Hedges (I^2H), and g index by Glass (I^2G).

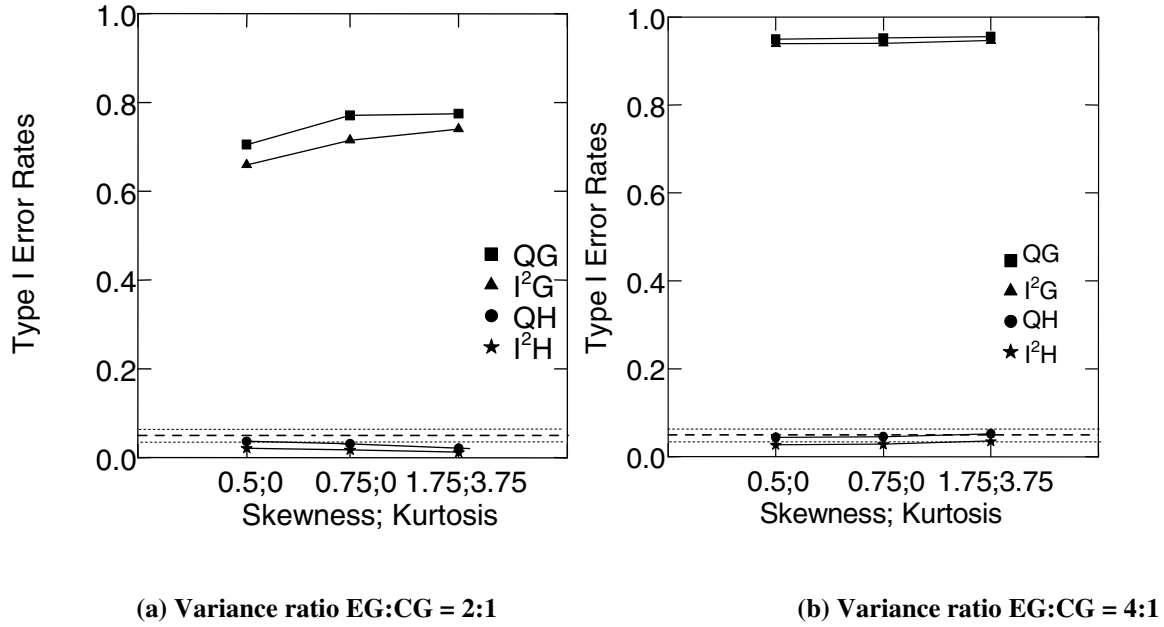


Figure 5. Type I error rates when non-normality and heterogeneous variances for experimental and control groups are assumed as a function of the levels of skewness and kurtosis, using a variance ratio of: (a) EG:CG= 2:1 and (b) EG:CG= 4:1, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 using d index by Hedges (I^2H), and g index by Glass (I^2G).

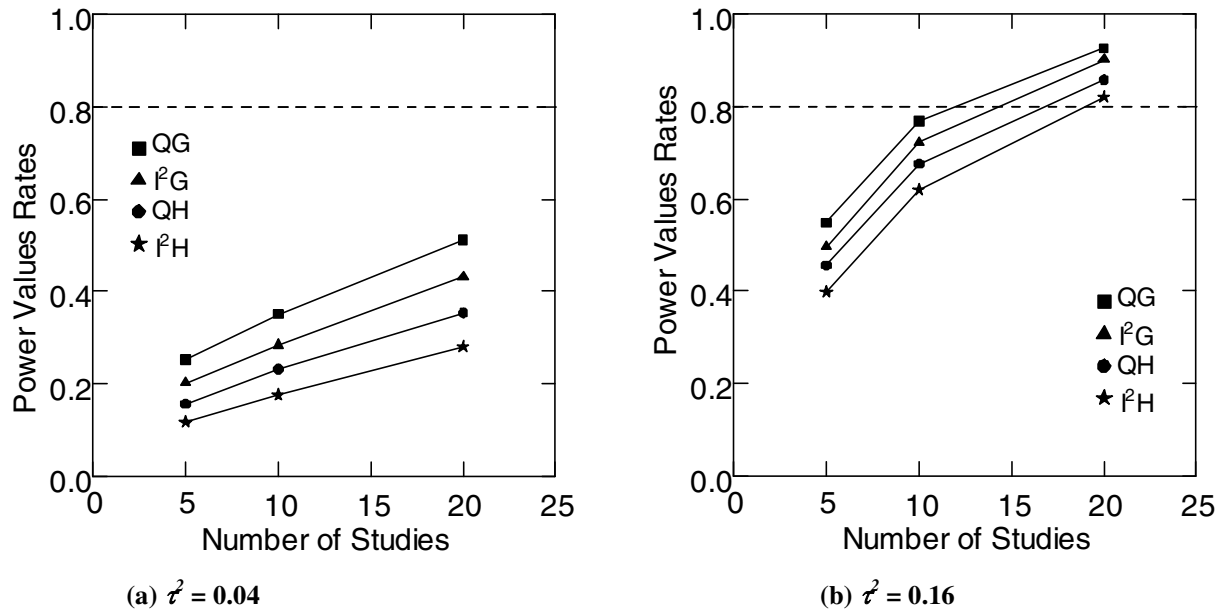


Figure 6. Power values rates when normality and homogenous variances for experimental and control groups are assumed as a function of the number of studies when: (a) $\tau^2 = 0.04$, and (b) $\tau^2 = 0.16$, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 using d index by Hedges (I^2H), and g index by Glass (I^2G).

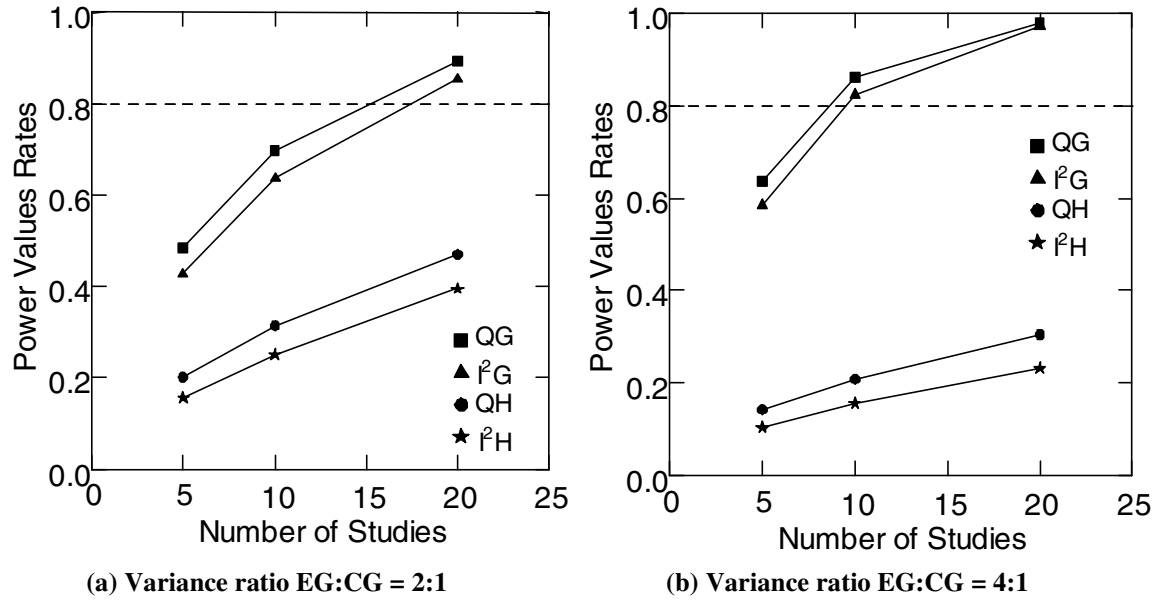


Figure 7. Power values rates ($\tau^2 = 0.08$) when normality and heterogeneous variances for experimental and control groups are assumed as a function of the number of studies when: (a) ratio EG:CG = 2:1 (b) ratio EG:CG = 4:1, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 using d index by Hedges (I^2H), and g index by Glass (I^2G).

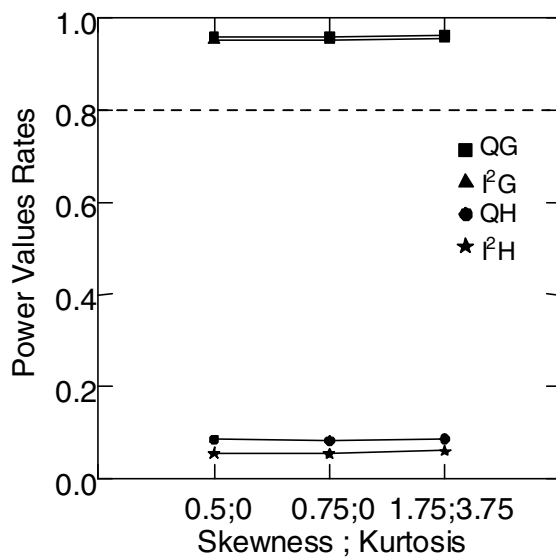
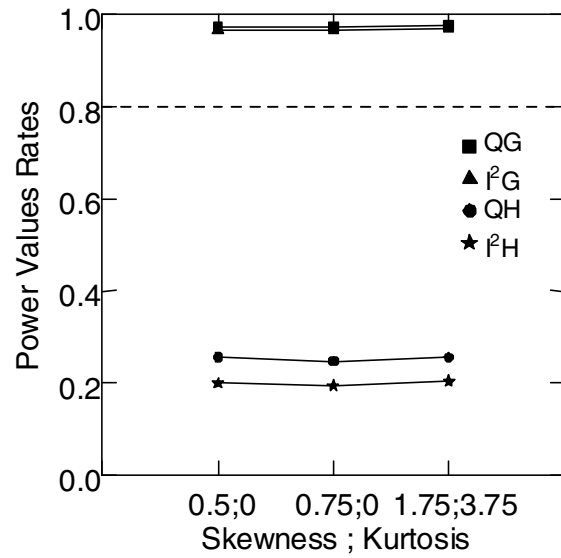
(a) $\tau^2 = 0.04$ (b) $\tau^2 = 0.16$

Figure 8. Power values rates when no normality and heterogeneous variances for experimental and control groups (ratio EG:CG = 2:1) are assumed as a function of levels of skewness and kurtosis when: (a) $\tau^2 = 0.04$ y (b) $\tau^2 = 0.16$, for the Q statistic using d index by Hedges (QH), and g index by Glass (QG), and for the confidence interval of I^2 using d index by Hedges (I^2H), and g index by Glass (I^2G).